J. Fluid Mech. (1983), *vol.* 137, *pp.* 447–456
Printed in Great Britain

The structure of the large eddies in fully developed turbulent shear flows. Part 2. The plane wake

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(Received 27 July 1982 and in revised form 9 February 1983)

A set of measurements using arrays of hot-wire anemometers has been performed in the fully developed turbulent wake of a circular cylinder. The data were digitized, recorded on magnetic tape, and processed using the pattern recognition technique described in Part 1 (Mumford 1982), to yield ensemble averages of the streamwise component of the velocity fields of the large eddies in the flow.

The results indicate that the large-scale structures in the turbulent wake are predominantly the inclined 'double-roller' vortices described by Grant (1958). These eddies consist of two contrarotating roller-like vortices with parallel axes displaced in the spanwise direction and approximately aligned with the direction of the strain associated with the mean velocity gradient. It was found that the structures are often confined to either side of the wake centreplane, rather than extending over the entire thickness of the turbulent region. In addition, eddies of similar type tended to occur in pairs or longer groups with their centres separated in the stream direction.

1. Introduction

The existence of a set of organized structures or large eddies within the turbulence of the fully developed plane wake was originally proposed by Townsend (1956). On the basis of an extensive series of velocity-correlation measurements, Grant (1958) suggested that these structures were of two types:

(1) the 'double-roller' eddies;

(2) the 'mixing jets' or entraining eddies.

The latter consisted of spanwise vortices, having circulation in the (x, z)-plane (the coordinate system is shown in figure 1), usually occurring in small groups, within which the centres of the individual structures are separated in the stream (X) direction, and have approximately the same Z-coordinate. A detailed description of the properties of these eddies has been given by Townsend (1979). In addition, Keffer (1965) and Kawall & Keffer (1981) have investigated the corresponding structures in the case of a plane wake subjected to a plane straining motion.

The form of the double-roller eddies found by Grant is depicted in figure 2 BB$\uparrow$. In this arrangement, the individual rollers have axes in the (x, z)-plane at angles of $\mathrm{sgn}\,(Z) \times 45°$ to the x-axis, and are combined as contrarotating pairs with axes separated in the y-direction. The flow patterns within the rollers took the form of a simple circulation parallel to the (x, y)-plane, having no w-velocity component. While the structures were portrayed as being of the BB$\uparrow$ type (upstream at the centre), measurements confined to the double velocity correlations provide no means of distinguishing between this type and the BB$\downarrow$ (downstream at the centre), and no reason for preference was indicated. By assuming a particular form for the eddy velocity fields, containing suitably adjusted constants, Grant obtained good agreement with his measurements, although the insensitivity of the correlations to the precise details of the eddy velocity distributions would imply that such agreement is dependent only on the gross features of the model.

Such methods of extracting the required structure from the experimental data are not entirely satisfactory, and a more formal alternative was proposed by Lumley (1967). This procedure employed a technique known as the proper orthogonal decomposition (or Karhunen–Loeve expansion), described by Loeve (1955), and required that the large eddies should be identified with the eigenfunctions of the correlation matrix. This enabled the forms of the eddy velocity fields to be uniquely defined and, subject to practical limitations, determined from the measured correlations. The application of this method to Grant's experimental data was described by Payne & Lumley (1967). Their computation yielded a first (most energetic) eigenfunction having some properties in common with figure 2 BB↓, although there were a number of significant differences. In particular, the roller axes were almost perpendicular to the wake centreplane, while the internal flow patterns were inclined to the (x, y)-plane (except at small $|Z|$), such that, towards the outer ends of the rollers, they were approximately normal to the $\operatorname{sgn}(Z) \times 45°$ directions (shown in figure 2 as the directions of the roller axes). In addition, there was apparently some reason for believing that the double rollers were of the BB↓ type, rather than the BB↑.

In the present work, data from arrays of hot-wire probes have been analysed, using the pattern-recognition technique described in Part 1 (Mumford 1982), to obtain a more detailed picture of the roller eddies.

2. The plane wake: experimental arrangement and instrumentation

The wake was generated by a 9.5 mm diameter circular cylinder spanning the upstream end of the working section of an open-return wind tunnel. The working section was 0.45 m square and 1.9 m long, and had a background (freestream) fluctuation intensity of approximately 0.2 % of the mean velocity. The tunnel speed was normally set at 10.3 m/s, giving a Reynolds number based on cylinder diameter of approximately 7×10^3. The coordinate system used in the experiments, and the scaling factors for the various measured quantities are illustrated in figure 1. The velocities will be denoted by (U, V, W), with mean values $(\overline{U}, \overline{V}, \overline{W})$, and fluctuations

$$(u, v, w) = (U, V, W) - (\overline{U}, \overline{V}, \overline{W}).$$

The correlation function for arbitrary variables p and q is defined by

$$R_{pq}(\boldsymbol{R}, \boldsymbol{r}, t) = \frac{\overline{p(\boldsymbol{R}, T)\, q(\boldsymbol{R}+\boldsymbol{r}, T+t)}}{[\overline{p(\boldsymbol{R}, T)^2}\ \overline{q(\boldsymbol{R}+\boldsymbol{r}, T)^2}]^{\frac{1}{2}}}$$

Since the small defect plane wake is approximately self-preserving (Townsend 1956), this function will be written as $R_{pq}(Z^*, \boldsymbol{r}^*, t^*)$.

The measurements were performed at a downstream distance (X) of 1.7 m (178 cylinder diameters), where the mean-velocity defect on the wake centreplane $U_1(X)$ was 8 % of the freestream velocity U_0, and the half-width of the wake $Z_0(X)$ was 35 mm. The eight hot-wire probes were arranged in a uniformly spaced row parallel to either the Y- or Z-directions. Data were recorded for probe spacings of both $0.57Z_0$ and $0.285Z_0$ for the Z-direction, and for the Y-direction at various values of Z.

The signal acquisition and processing equipment was similar to that described in Part 1. For measurements in the fully developed wake, linearization of the anemometer

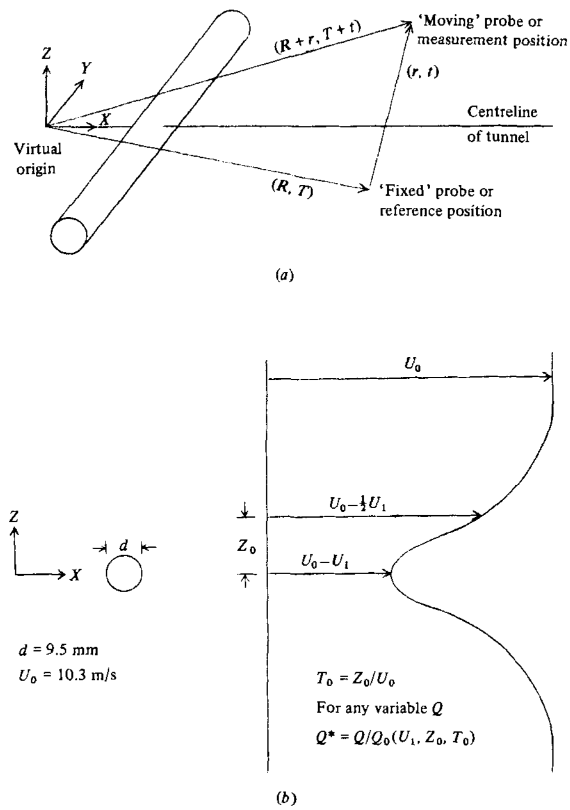

FIGURE 1. (*a*) Coordinate system and (*b*) scaling factors for distances and velocities for the plane wake.

output voltages is unnecessary, since the turbulent intensity relative to the mean velocity is quite small. The anemometer outputs were therefore digitized with 8-bit resolution, following the application of suitable offset and gain. The eight channels were simultaneously sampled at the rate of 1000 per second, and the processing was performed on records of typically 32K samples per channel. When analysing the tapes, a linear relation was assumed for the dependence of the digitized voltages on the velocity fluctuations u.

3. The large eddies in the plane wake

While the measurements of Grant (1958) provided convincing evidence for the presence of double-roller structures of the type shown in figure 2 BB $\uparrow$ or $\downarrow$, the difficulty of deducing the form of the eddy velocity fields from the measured correlations inevitably leads to a degree of uncertainty in the results. Even under the most favourable circumstances, when all the contributing structures are identical, the resolution available is barely adequate, and deteriorates considerably in the more realistic case where the structures are of similar type but cover a range of sizes. It has been found (Townsend 1979), for example, that from measurements of R_{uu} alone it is hardly possible to distinguish between single rollers (figure 2 A) and double rollers (figure 2 B $\uparrow$ or $\downarrow$).

In Grant's work, the existence of the double structures was inferred largely from

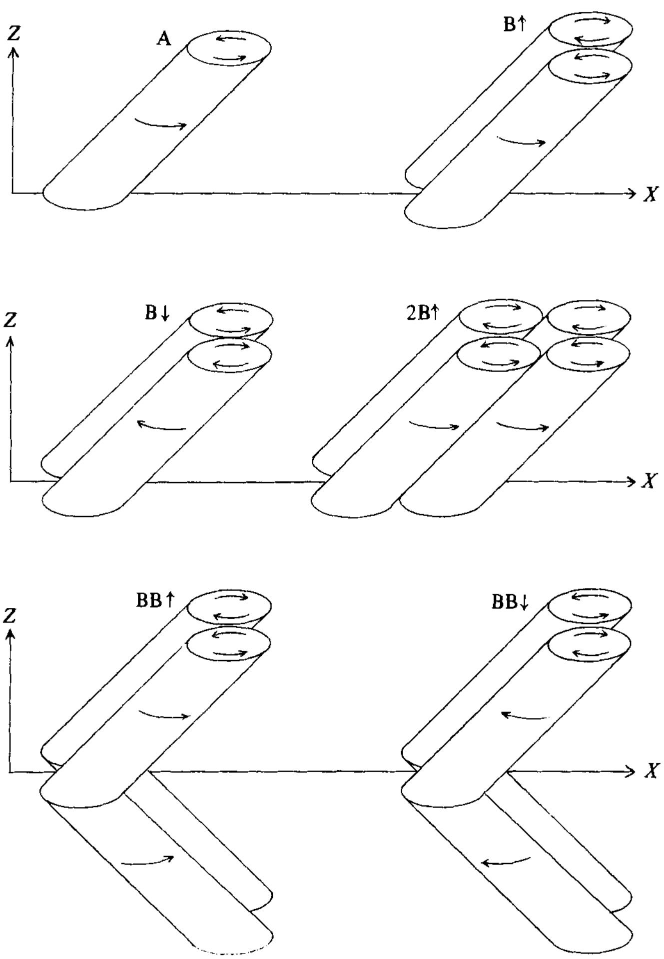

FIGURE 2. The strainwise roller eddies.

the form of the $R_{vv}(Z, \mathbf{r}, 0)$† correlation. The question arises, however, of whether it is possible, on this basis, to distinguish between the 'complete' structures (figure 2 BB ↑ or ↓) and a mixture of these structures and their component parts (figure 2 A, B ↑ or ↓). Specifically, while the form of $R_{vv}(Z,(x,0,z),0)$ indicates that at least some of the rollers extend across the full thickness of the wake, the decrease in the amplitude of this function with increasing $|z|$ (particularly noticeable at $z = -Z$) suggests that many of the structures may be of more limited extent in the Z-direction, as shown in figure 2 B ↑ or ↓. In addition, while the form of $R_{vv}(Z,(x,y,0),0)$ indicates the presence of double rollers, it would be difficult to exclude the possibility of a contribution from unpaired structures of the type shown in figure 2 A.

Using the data-processing techniques described in Part 1, some progress can be made in distinguishing between the various possibilities outlined above. The data obtained in the (y,t)-plane were initially analysed using a test pattern of the form

$$u(x,y) = k_y\, y \exp\left(-\tfrac{1}{2}k_x^2\, x^2 - \tfrac{1}{2}k_y^2\, y^2\right),$$

representing a single roller (figure 2 A) velocity field, with longitudinal and transverse

† $R_{33}(Y, \mathbf{r})$ in Grant's notation.

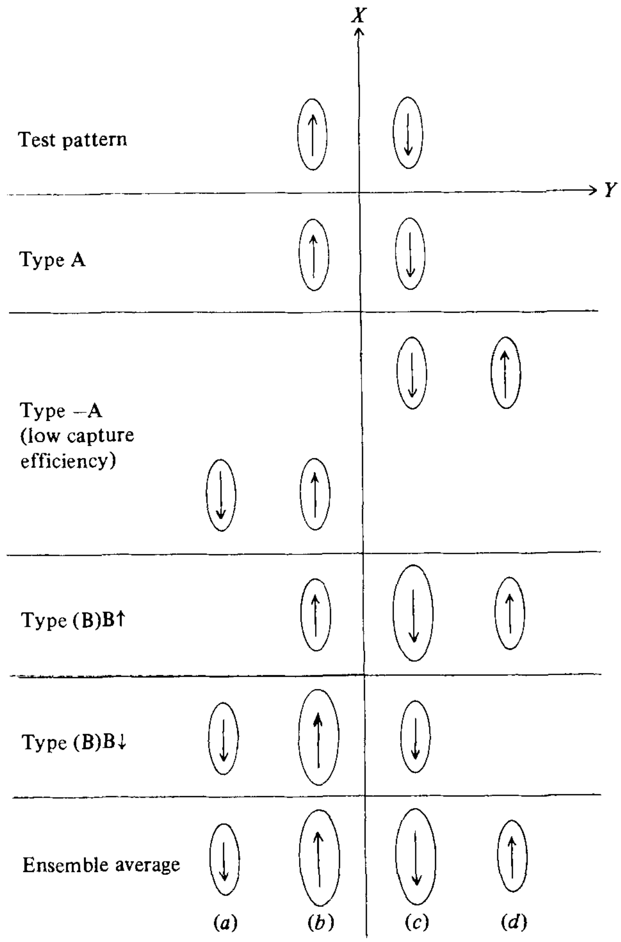

FIGURE 3. The transverse (Y) alignments and resulting ensemble average obtained from a single-roller test pattern for a flow containing both single- and double-roller eddies.

scales determined by k_x and k_y. With the recognition programme set to extract the velocity fields yielding the largest convolutions with the test pattern (as opposed to the largest moduli of the convolutions, which had been used in the case of the jet), the expected result of using an initial pattern of this type on a flow containing a mixture of single rollers and double rollers of both types (↑ and ↓) is illustrated in figure 3. While the diagram indicates the possibility of a contribution from single rollers having circulation in the opposite sense to that of the test pattern, the probability of a measurable contribution would be small, due to the relatively large amplitude that such structures would require for inclusion in the average. Consequently, the generation of additional velocity fields in positions (d) and (a) in the ensemble average in figure 3 would indicate the presence of double rollers of types (B)B↑ and (B)B↓ respectively.†

The experimental result from data obtained at $Z^* = 1.0$ is shown in figure 4. In addition to the two subsidiary velocity fields generated either side of the original pattern, the ensemble average also contains local maxima displaced from the test pattern in the stream direction. The amplitudes of the four velocity peaks

† In the discussion of the results from the (x, y)-plane, '(B)B' will be used to denote 'BB or B'.

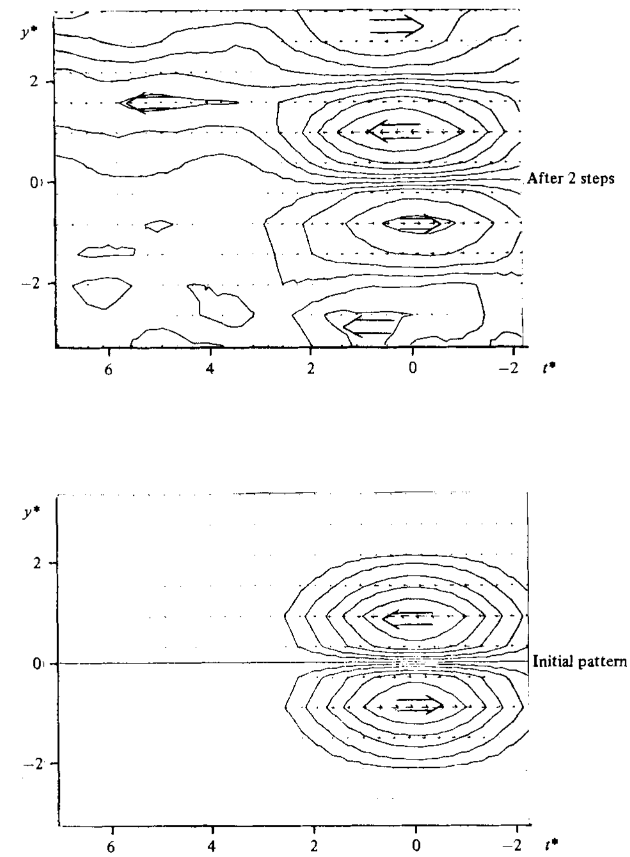

FIGURE 4. Patterns in the (x, y)-plane at $Z^* = 1.0$ (type A test pattern: see text and figure 3).

corresponding to (*a*), (*b*), (*c*) and (*d*) in figure 3 give some indication of the relative contributions from the three types of eddies A, (B)B$\uparrow$ and (B)B$\downarrow$ in figure 2, although a quantitative decomposition would require rather more information than is available. It seems that the type-(B)B$\uparrow$ double rollers make a considerably larger contribution than the type (B)B$\downarrow$, and that there is also a significant contribution from the type-A (unpaired) structures. From visual inspection of a large number of instantaneous velocity profiles, Savill (1979) concluded that the velocity patterns corresponding to types (B)B$\uparrow$ and (B)B$\downarrow$ occurred with approximately equal frequency. The larger contribution obtained from the type-(B)B$\uparrow$ structures would therefore suggest that their typical amplitudes are somewhat larger than those of the type (B)B$\downarrow$.

In order to determine the detailed structure of the double-roller velocity fields, the data obtained at $Z^* = 1.0$ were reanalysed using an initial pattern of the form

$$u(x,y) = (k_y^2\, y^2 - 1)\exp\left(-\tfrac{1}{2}k_x^2\, x^2 - \tfrac{1}{2}k_y^2\, y^2\right),$$

corresponding to a type-(B)B$\uparrow$ double roller. The result is shown in figure 5. The velocity distribution within the ensemble average is such that the downstream maxima in the 'sidelobes' have half the magnitude of the upstream central maximum. For a fixed inclusion limit in the selection procedure, the averaged peak velocities within the structures relative to the local turbulent intensity $[\overline{u^2(Z^*)}]^{\frac{1}{2}}$ were somewhat larger than in the case of the jet (see Part 1). Individual (B)B$\uparrow$ structures

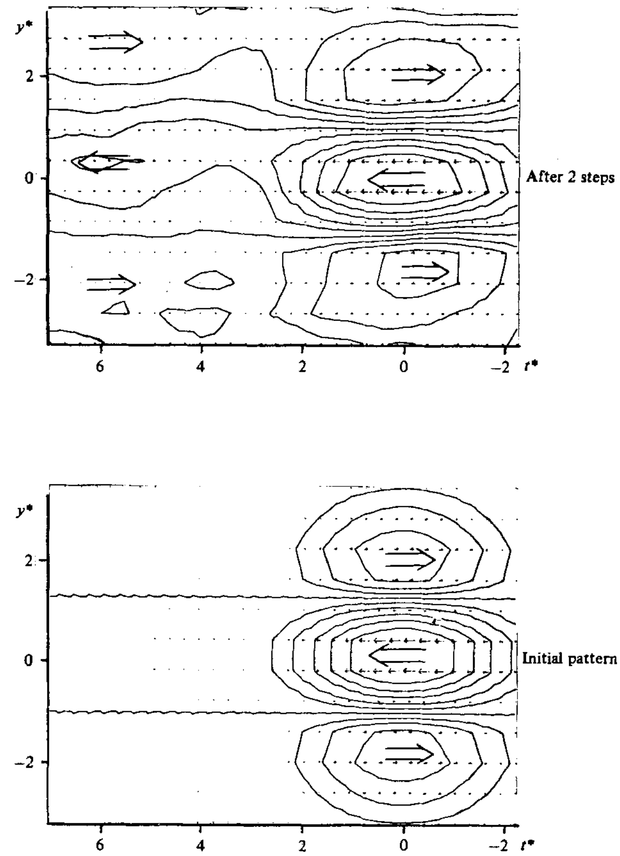

FIGURE 5. Patterns in the (x, y)-plane at $Z^* = 1.0$ (type (B)B↑ test pattern).

with peak amplitudes greater than $1.8[\overline{u^2(Z^*)}]^{\frac{1}{2}}$ occurred quite frequently, and this was sufficient to ensure that the probability of the inclusion of either type-A or type-(B)B↓ eddies in the ensemble was fairly small. It should be noted, however, that the application of a double-roller test pattern to a flow which contained *only* single-roller structures would yield an ensemble average similar to the result actually obtained.

As indicated in Part 1, the structures included in the ensemble average would be expected to exhibit a range of properties. In addition to variability of size, the double rollers in the wake may have a spread of possible angular orientations in the (x, y)-plane, in the sense that the two roller axes of an individual structure may have a range of possible x-separations (centred on zero). This would tend to yield an average which is elongated in the stream direction by comparison with the result that would have been obtained from an equivalent set of structures having exact alignment.

The additional velocity maxima generated outside the original pattern, and separated from it in the stream direction, demonstrate a further degree of organization of the structures within the flow. While this feature could be generated by suitably arranged combinations of type-(B)B↑ double rollers with type-A single rollers, cursory inspection of the velocity traces revealed the frequent occurrence of pairs of type-(B)B↑ velocity fields, consistent with the arrangement denoted by 2B↑ in figure

2, and with a streamwise spacing corresponding to figure 5. A more thorough inspection by Savill (1982) revealed the presence of longer sequences of double rollers, nB$\uparrow$ or $\downarrow$, with centres having approximately the same Y-coordinate, and separated in the X-direction with a spacing consistent with figure 5. Values of n up to 5 were observed. Close examination of Grant's (1958) correlation measurements indicates that, at large x, the measured values of $R_{uu}(Z^*,(x,0,0),0)$ lie above the fitted curve (which was based on a (B)B$\uparrow$ velocity field), and $R_{vv}(Z^*,(x,0,0),0)$ exhibits a second change of sign.† While the magnitudes of these features are too small to allow any inference to be made, they are consistent with the presence of pairs (or longer groups) of double rollers arranged as shown in figure 2 2B$\uparrow$ or $\downarrow$.

An attempt to find the velocity distribution within the type-(B)B$\downarrow$ double rollers at $Z^* = 1.0$ using an initial pattern of the form

$$u(x,y) = (1-k_y^2\,y^2)\exp\left(-\tfrac{1}{2}k_x^2\,x^2-\tfrac{1}{2}k_y^2\,y^2\right),$$

proved unsuccessful. The selected velocity patterns contained not only the required structures, but also the higher-intensity type-(B)B$\uparrow$ structures, offset in the Y-direction such that their centres were aligned with either of the sidelobes on the original pattern. Consequently, it was not possible to obtain any precise information from the ensemble average, although there was some indication that the velocity distribution within the type-(B)B$\downarrow$ structures may be different (apart from having the opposite sign) to that found for the type-(B)B$\uparrow$ structures (figure 5).

A similar problem occurred in the analysis of the data from the (y,t)-plane at $Z^* = 0.0$. It was found that the typical amplitudes of the type-(B)B$\uparrow$ double rollers relative to $[\overline{u^2(Z^*)}]^{\frac{1}{2}}$ were smaller on the wake centreplane than at $Z^* = 1.0$, and were comparable to those of the type (B)B$\downarrow$. Consequently, the ensemble averages obtained using test patterns of types (B)B$\downarrow$ and $\uparrow$ contained not only the required structures but also contributions from eddy types (B)B$\uparrow$ and $\downarrow$ respectively, and probably type A. The statistics of the pattern selection procedure indicated that the occurrence frequencies of the double rollers were smaller at $Z^* = 0.0$ than at $Z^* = 1.0$, implying that a proportion of the structures must be confined to either side of the wake centreplane as shown in figure 2 B$\uparrow$ or $\downarrow$, rather than continuing across the entire turbulent region as shown in figure 2 BB$\uparrow$ or $\downarrow$.

The data obtained in the (z,t)-plane were analysed using initial patterns of the form

$$u(x,z) = \exp\left(-\tfrac{1}{2}k_x^2(x-sz)^2\right) \quad (z<0),$$
$$= 0 \quad (z>0),$$

representing a slice parallel to the (x,z)-plane through a roller confined to the region $z<0$, with its axis at an angle of $180°-\tan^{-1}(1/s)$ to the X-axis, and with a longitudinal scale determined by k_x. As in the case of the jet, it was assumed that the structures are 'locked' in position in the Z-direction, so that a test pattern 7 probe spacings wide was used, allowing no transverse alignment.

Some of the results for $s = 0$ are shown in figure 6, and indicate that the roller axes are concentrated at angles of approximately sgn $(Z)\times 45°$ to the X-axis, and are thus aligned with the rate of strain associated with the mean velocity gradient, in agreement with the result obtained by Grant (1958). The sharpness of the corners on the second and third velocity contours (counting from the maximum) where they cross the wake centreplane provides justification for the assumption that the structures have only a small range of possible positions in the Z direction. The velocity peak generated outside the original pattern in the region $Z<0$ corresponds to the

† $R_{11}(Y,(r_1,0,0))$ and $R_{33}(Y,(r_1,0,0))$ in Grant's paper.

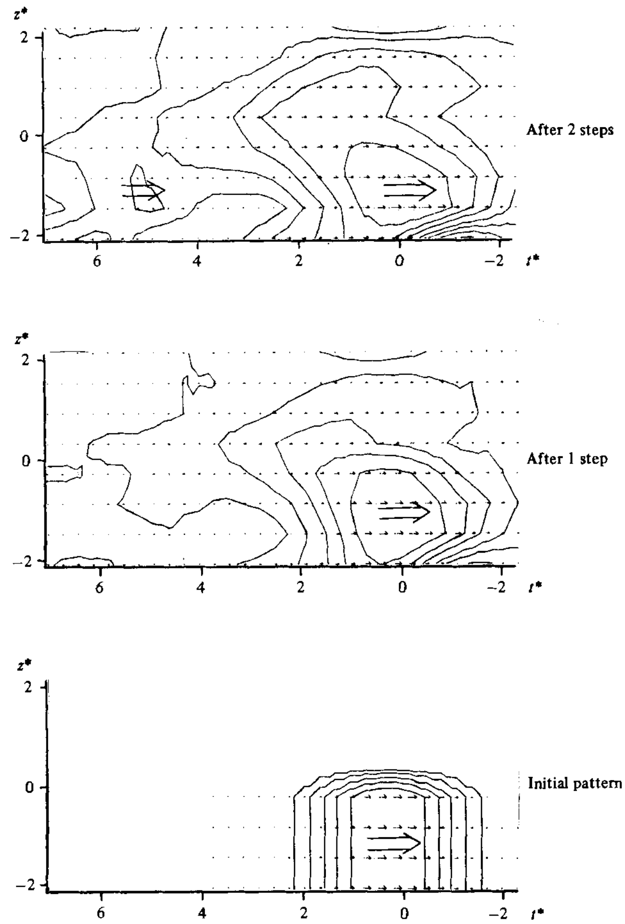

FIGURE 6. Patterns in the (x, z)-plane $(Z^* = 0)$.

streamwise pairing (or grouping) found in the (x, y)-plane, as shown in figure 2 2B† (for the region $Z > 0$).

In estimating the proportion of the rollers that cross the wake centreplane, it should be remembered that the selection procedure for the velocity patterns is weighted towards those covering the largest area of the test pattern. Consequently, following the first step of the iteration, the selection of patterns for the second step will be weighted in favour of the double-sided (type-BB) structures, giving an overestimate of their contribution relative to the type-B structures. This difficulty can be overcome by resetting the velocity field in the region $Z > 0$ (figure 6) to zero after the first programme step, so that an unbiased selection should be obtained for the second step. The result suggested that approximately 30 % of the roller eddies continue for an appreciable distance beyond the wake centreplane.

4. The relation between the double-roller eddies and the mixing jets

The possibility that the two types of large-scale structure within the wake are interconnected has been discussed by Grant (1958) and by Savill (1979). It was suggested that the mixing jets joined the two halves of the double-roller structures at their outer ends, although the evidence for this arrangement was somewhat indirect. Townsend (1979) has shown that the mixing jets occur in groups of between 3 and 5 individual structures, exhibiting fairly uniform streamwise spacing within any particular group. In addition, estimates were obtained for the range of possible streamwise spacings, corresponding to different stages in a growth and decay cycle, covered by the eddy groups. The result obtained in §3, suggesting the streamwise grouping of the double-roller eddies, with a spacing within the range found by Townsend for the spacing of the mixing jets, represents somewhat stronger evidence that the two types of structure are connected, and it seems very probable that one type is generated as a consequence of the presence of the other.

Townsend (1966) has suggested a mechanism for the production of groups of mixing jets at the interface between turbulent and irrotational fluid. One possibility, therefore, is that the strainwise double rollers are formed from the spanwise mixing jets by the 'kinking-and-stretching' instability to which spanwise vortices in a shear flow are susceptible (see Hunt 1977). Such a mechanism, acting on a mixing jet with circulation in the same sense as the local mean-velocity field, would produce a B$\downarrow$ velocity field if the kinking motion were towards the wake centreplane. Similarly, a B$\uparrow$ velocity field would be produced if the same structure were subjected to kinking away from the wake centreplane, yielding an arrangement similar to the type described by Grant and by Savill. The direction in which such an instability could be sustained must depend firstly on the distance of the mixing-jet centres from the wake centreplane, which is dependent on their 'age' in the growth–decay cycle (Townsend 1979), and secondly on their convection velocity. The determination of the precise details of the mechanism would therefore appear to require further measurements, possibly involving a sufficiently large number of probes for the simultaneous detection of the two types of structure. It seems, however, that the production of a double-sided (type-BB) structure could only be initiated by the inward kinking of a mixing jet with its centre situated at suitably large $|Z|$.

The author would like to thank Dr A. M. Savill and Dr A. A. Townsend for their advice, and for allowing the use of their experimental data.

The work was supported by the Science and Engineering Research Council, and computing facilities were provided by the University of Cambridge Computing Service.

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